GLENWOOD

Student No. | | | | | | | | | |

Student Name: ______________________________

Teacher Name: ______________________________

2020 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Approved calculators may be used
- A reference sheet is provided
- In Questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 2 – 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 9 – 14)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I
10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

1. What is the derivative of $\cos^{-1}\left(\frac{x}{5}\right)$?

(A) $-\frac{1}{\sqrt{25-x^2}}$

(B) $\frac{1}{\sqrt{25-x^2}}$

(C) $-\frac{1}{\sqrt{5-x^2}}$

(D) $\frac{1}{\sqrt{5-x^2}}$

2. What is the rate of change of the volume of a cube, when the side length of the cube is 4 cm and the surface area is increasing at $2\text{ cm}^2\text{s}^{-1}$?

(A) $1\text{ cm}^3\text{s}^{-1}$

(B) $2\text{ cm}^3\text{s}^{-1}$

(C) $8\text{ cm}^3\text{s}^{-1}$

(D) $18\text{ cm}^3\text{s}^{-1}$

3. An examination consists of 30 multiple-choice questions, each question having five possible answers. A student guesses the answer to every question. Let *X* be the number of correct answers. What is ()?

(A) 5

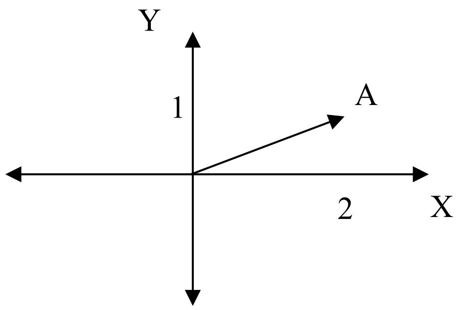
(B) 6

(C) 9

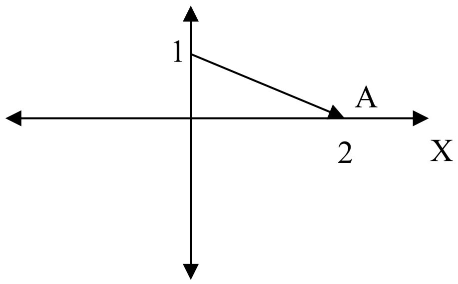
(D) 15

4. The position vector $\vec{OA} = \binom{2}{1}$ is best represented by

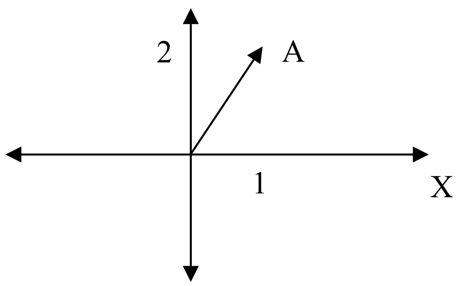
(A)



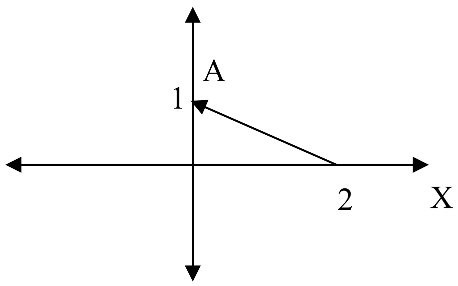
(B)



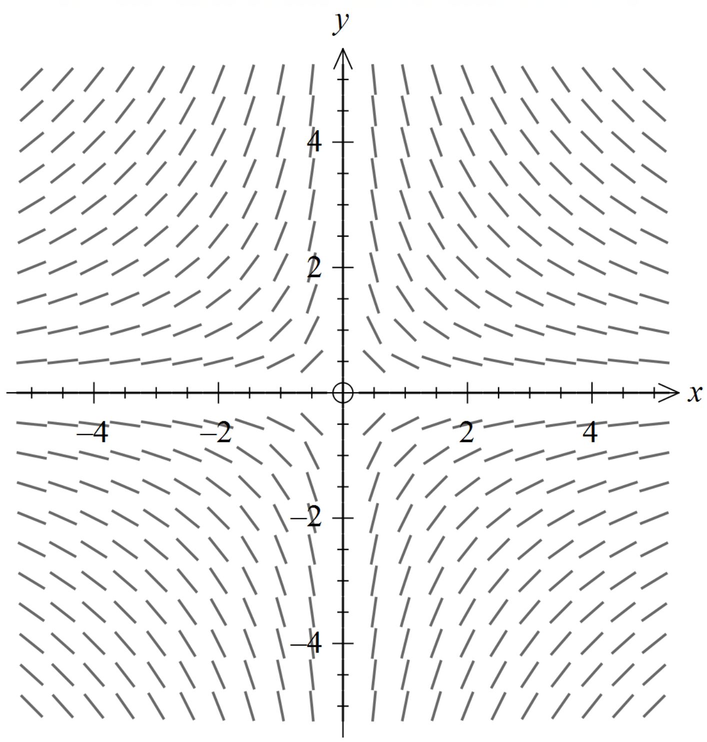
(C)



(D)



5. The slope field for a first order differential equation is shown.



What could be the differential equation represented?

(A) $\frac{dy}{dx} = -\frac{x}{y}$

(B) $\frac{dy}{dx} = \frac{x}{y}$

(C) $\frac{dy}{dx} = -\frac{y}{x}$

(D) $\frac{dy}{dx} = \frac{y}{x}$

6. Which of the following is true for the vector $\vec{OB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

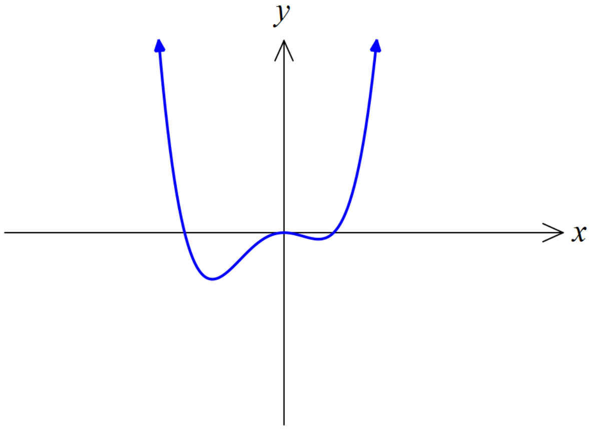
(A) magnitude $= 5$ and direction is 53^0

(B) magnitude $= 25$ and direction is 37^0

(C) magnitude $= 5$ and direction is 37^0

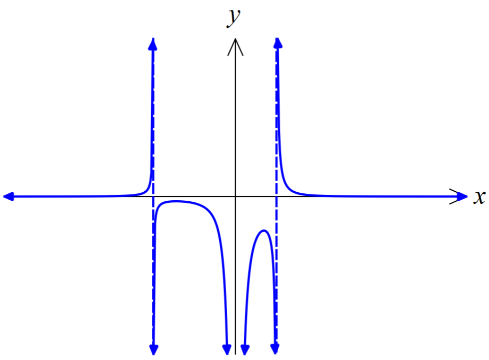
(D) magnitude $= 7$ and direction is 37^0

7 The diagram shows the graph of $f(x)$.

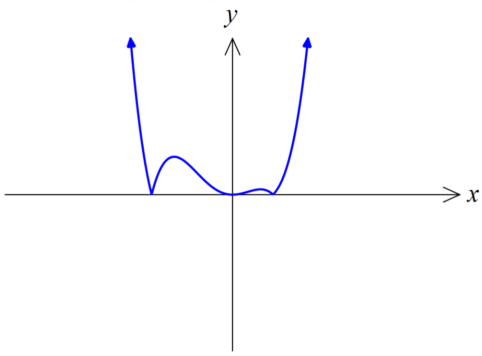


Which graph best represents the function $y = |f(x)|$?

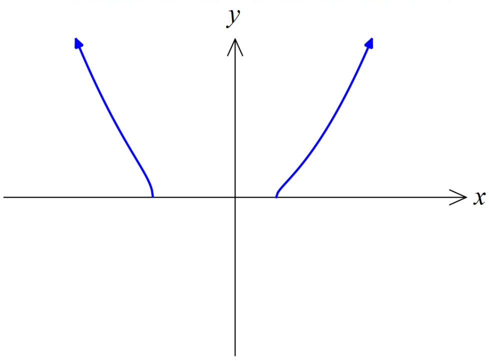
(A)



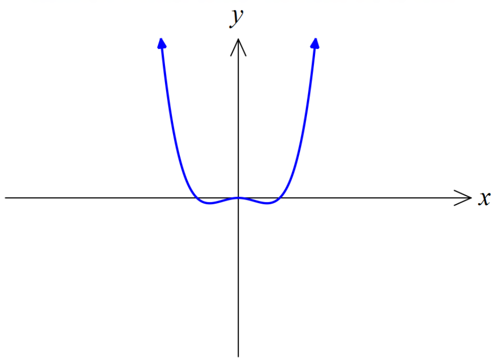
(B)



(C)



(D)



8. Which of the following is **NOT** equal to the area of the region bounded by the curves $y=\sin x$ and $y=\cos x$ between $x=0$ and $x=\dfrac{\pi}{4}$?

(A) $-\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x-\sin x \, dx$

(B) $\displaystyle\int_{\frac{-\pi}{4}}^{0} \cos x-\sin x \, dx$

(C) $\displaystyle\int_{\frac{-3\pi}{4}}^{\frac{-\pi}{2}} \cos x-\sin x \, dx$

(D) $\displaystyle\int_{\pi}^{\frac{5\pi}{4}} \cos x-\sin x \, dx$

9. $\cos 3\theta+\sin 3\theta$ in the form $R\cos(3\theta-\alpha)$ and α is in degrees is

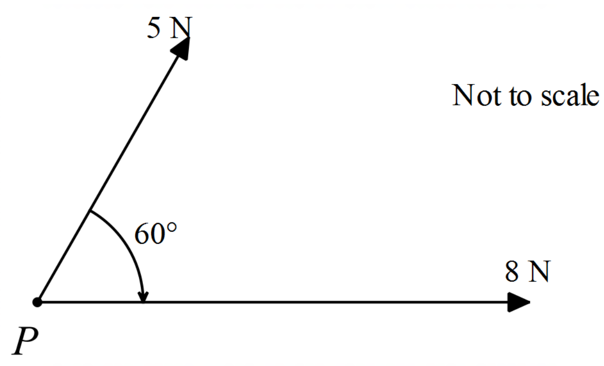
(A) $\sqrt{2}[\cos(3\theta-45)]$

(B) $\sqrt{2}\,[\cos(3\theta+45)]$

(C) $\frac{\sqrt{3}}{2}[\cos(3\theta-45)]$

(D) $\frac{\sqrt{3}}{2}[\cos(3\theta+45)]$

10. Forces of magnitude 8 N and 5 N act on a particle P. The angle between the directions of the two forces is $60°$ as shown in the diagram.



Which of the following is the correct magnitude and direction of the resultant force acting on P?

(A) 11.36 N, $22°25'$ to the horizontal

(B) 11.36 N, $67°35'$ to the horizontal

(C) 12.58 N, $22°25'$ to the horizontal

(D) 12.58 N, $67°35'$ to the horizontal

BLANK PAGE

Section II

60 marks

Attempt Questions 11 – 14.

Allow about 1 hour and 45 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet.

a) Solve $\frac{x+2}{x-5} \geq 0$. **2**

b) The polynomial equation $x^3 - 5x^2 + x + 3 = 0$ has roots α, β and γ Find $\alpha^2 + \beta^2 + \gamma^2$ **2**

c) i) Write an expression for $\sin 5x \sin x$ in terms of $\cos 4x$ *and* $\cos 6x$ **2**

ii) Hence, find $\int_0^{\frac{\pi}{4}} \sin 5x \ \sin x \ dx$. **2**

d) Weather records suggest that in the town of Mathsville, an average of 5 days out of of 30 in the month of April will be wet. **3**

If X represents the number of wet days in April, such that $X \sim \text{Bin}\left(30, \frac{1}{6}\right)$, find $P(X \geq 2)$.

e) The column vector notation of four vectors is shown below.

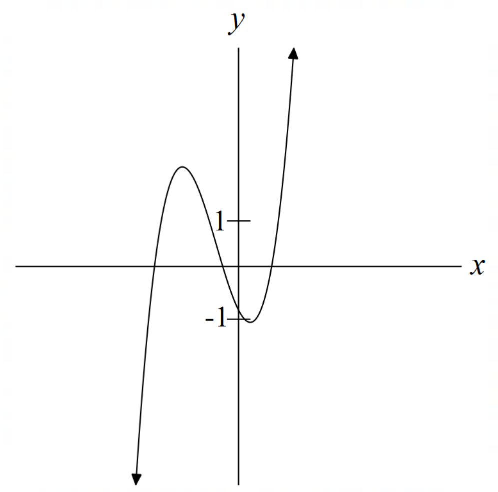
$P = \begin{pmatrix} -8 \\ -8 \end{pmatrix}$ $Q = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$ $R = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ $S = \begin{pmatrix} -5 \\ 7 \end{pmatrix}$. Find

i) $\overrightarrow{PQ}$ **1**

ii) $-\overrightarrow{PQ} - \overrightarrow{RS}$ **1**

Question 11 continues on next page

f) The graph of $f(x)$ is shown in the diagram. **2**



Sketch the graph of $y^2 = f(x)$.

End of Question 11

Question 12 (13 marks) Start a new writing booklet.

a) A game is played in which players throw netballs, aiming to get them through the hoop.

The players are given two options in how they play the game.

Option 1: Players take 2 attempts and win a prize if at least one of the balls goes through.

Option 2: Players take 3 attempts and win a prize if at least two of the balls go through.

Karen decides to play.

The probability that she will get a ball through on any attempt is p, where $p > 0$.

i) Show that the probability that Karen wins if she takes Option 1 is $2p - p^2$. **1**

ii) Show that the probability that Karen wins if she takes Option 2 is $3p^2 - 2p^3$. **1**

iii) Karen has calculated that she is 3 times as likely to win by choosing Option 1 over Option 2. Find the value of p. **2**

b) Consider the statement $P(n)$:
$2^0 + 2^1 + 2^2 + \cdots + 2^{n-1} = 2^n - 1$
and the following attempted proof of this statement by induction.

Proof:
Assume the statement is true for $n = k$. That is,
$2^0 + 2^1 + 2^2 + \cdots + 2^{k-1} = 2^k - 1 \quad (1)$

Next, we shall show it is true for $n = k + 1$ by noting that if
$2^0 + 2^1 + 2^2 + \cdots + 2^{k-1} + 2^k = 2^{k+1} - 1$
is true, then
$2^0 + 2^1 + 2^2 + \cdots + 2^{k-1} + 2^k = 2 \times 2^k - 1$
$2^0 + 2^1 + 2^2 + \cdots + 2^{k-1} + 2^k = 2^k + 2^k - 1$
Now subtracting 2^k from both sides of this equation, we have
$2^0 + 2^1 + 2^2 + \cdots + 2^{k-1} = 2^k - 1$
Which is true by statement (1). Therefore, by the principle of induction, the statement $P(n)$ is true.

i) Give two reasons why the given proof is incorrect and does not prove $P(n)$. **2**

ii) Provide a correct complete proof by induction of the statement $P(n)$. **2**

c)

i) Use the substitution $t = \tan\frac{x}{2}$ to show that $\text{cosec}x + \cot x = \cot\frac{x}{2}$. **2**

ii) Hence evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\text{cosec}x + \cot x)\,dx$. Answer is simplest exact form. **3**

End of Question 12

Question 13 (17 marks) Start a new writing booklet.

a) On a roulette wheel, there are 18 red numbers, 18 black numbers, and one green number. A ball is dropped onto the spinning wheel and lands on one of the numbers randomly. Each result is independent.

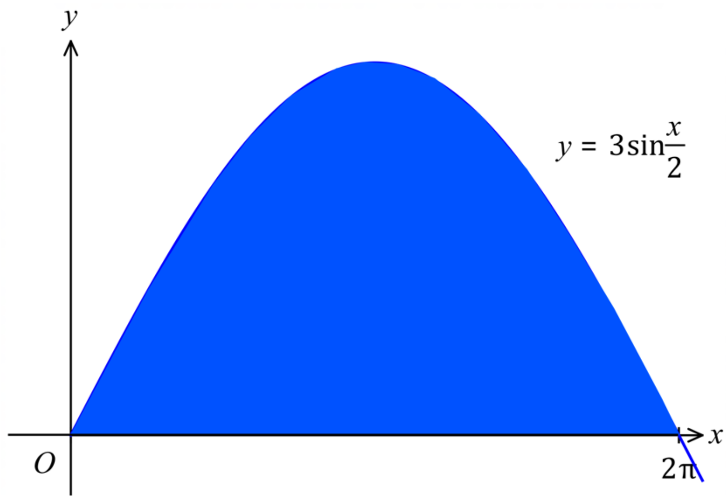
A gambler bets that the ball will land on any of the 18 black numbers.

i) Define the gambler's bet as a Bernoulli random variable X and give its mean and variance. **2**

ii) If the gambler makes the same bet five times, let the random variable Y be the number of times the gambler wins. Describe the distribution of Y, and give its mean and variance. **2**

iii) If the gambler makes the same bet five times, what is the probability he will win more times than he loses? Give your answer correct to three decimal places. **2**

b)



Shaded region is enclosed by the curve $y = 3\sin\frac{x}{2}, 0 \leq x \leq 2\pi$ and the x axis.

i) Find the area of the shaded region. **1**

ii) This region is rotated through 2π radians about the x-axis. Find the volume of the solid generated. **2**

Question 13 continues on next page

c) A tank contains 2,500 litres of water and 25 kg of dissolved salt. Fresh water enters the tank at a rate of 20 litres per minute. The solution is thoroughly mixed at all times and is drained from the tank at a rate of 15 litres per minute.

i) Using y for the amount of salt in the tank in kilograms (as a function of time), and t for time in minutes, show that the concentration of salt in the tank at time t can be given by **1**

$$C = \frac{y}{2500 + 5t}$$

ii) Explain why the rate of change of salt in the tank can by given by $y' = -15C$. **1**

iii) Find y, the amount of salt in the tank as a function of t. **4**

iv) If the tank has a capacity of 5,000 litres, how much salt is in the tank when it overflows? **2**

End of Question 13

Question 14 (15 marks) Start a new writing booklet.

a) For the differential equation $y' = \dfrac{6}{5x^2 + 4x - 1}$:

i) Show that $y' = \dfrac{5}{5x - 1} - \dfrac{1}{x + 1}$. **1**

ii) Find the solution to the differential equation; given that when $x = \dfrac{1}{2}$, $y = 3$. **2**

b) A golf ball is hit at a velocity of 110 m/s at an angle θ, to the horizontal.

The position vector s(t), from the starting point, of the ball after t seconds is given by

$s = 110t\cos\theta\, \underset{\sim}{\mathbf{i}} + (110t\sin\theta - 4{\cdot}9\,t^2)\underset{\sim}{\mathbf{j}}$

i) Using gravity of $9{\cdot}8\ \text{ms}^{-2}$ show that the maximum horizontal range of the ball is $\dfrac{12100\sin 2\theta}{9{\cdot}8}$ metres. **2**

ii) To ensure that the ball lands on the green, it must travel between 400 and 450 metres. What values of θ, correct to the nearest minute, would allow this to happen? **2**

iii) The golfer hits the ball directly towards the green with a velocity of 110 m/s. **2**

After 3.4 seconds of flight, at a point 8 metres above the ground, the ball hits a low flying TV drone. If it had not hit the drone or any other obstacles, would the ball have landed on the green?

c) Consider the function $y = \cos^{-1}(\sin x)$.

i) Find $\dfrac{dy}{dx}$. **1**

ii) What does your answer to part (a) tell you about stationary points for this function? **1**

iii) State the domain and range for this function. **2**

iv) Sketch the function over the domain $0 \le x \le 2\pi$ **2**

End of paper

BLANK PAGE

BLANK PAGE

GLENWOOD

Mathematics Extension 1

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Multiple Choice Answer Sheet

Full Name: Solutions

Teacher Name: ______________________

Completely fill the response oval representing the most correct answer.

1.	A ●	B ○	C ○	D ○
2.	A ○	B ●	C ○	D ○
3.	A ○	B ●	C ○	D ○
4.	A ●	B ○	C ○	D ○
5.	A ○	B ○	C ●	D ○
6.	A ○	B ○	C ●	D ○
7.	A ○	B ●	C ○	D ○
8.	A ○	B ●	C ○	D ○
9.	A ●	B ○	C ○	D ○
10.	A ●	B ○	C ○	D ○

Aryan → no name on 2nd booklet

Ruhi → Q13 started on Q12 booklet

Ashna → number her booklets not per question but total

Shuti, Shardian, Preeta & Ruhi & Praneet → did not number their bookets 1 of 1

Question 11

GLENWOOD

Name: ______________________

Teacher: ______________________

Question No.

☐ Year 11 ☐ Year 12

☐ Adv ☐ Ext 1 ☐ Ext 2

Question 11

a) $\frac{x+2}{x-5} \geq 0 \qquad x \neq 5$

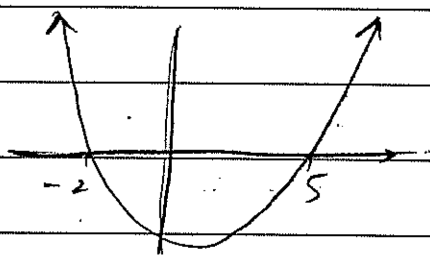
$\frac{(x+2)(x-5)^{2}}{x-5} \geq 0(x-5)^2$

$(x-5)(x+2) \geq 0$

$x^2-3x-10 \geq 0$

$(x-5)(x+2) \geq 0$

$x \leq 2 \qquad x > 5$



b) $\alpha+\beta+\gamma = 5$

$\alpha\beta+\alpha\gamma+\beta\gamma = 1$

$\alpha\beta\gamma = -3$

$\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$

$= (5)^2 - 2(1)$

$= 25 - 2 = 23$

c) $\sin 5x \sin x = \frac{1}{2}[\cos(5x-x) - \cos(5x+x)]$

$= \frac{1}{2}[\cos 4x - \cos 6x]$

(ii) $\int_0^{\pi/4} \sin 5x \sin x \, dx$.

$= \frac{1}{2}\int_0^{\pi/4} \cos 4x - \cos 6x$

$= \frac{1}{2}\left[\frac{\sin 4x}{4} - \frac{\sin 6x}{6}\right]_0^{\pi/4}$

$= \frac{1}{2}\left[\left(\frac{\sin \pi}{4} - \frac{\sin \frac{6\pi}{4}}{6}\right) - \left(\frac{\sin 0}{4} - \frac{\sin 0}{6}\right)\right]$

$= \frac{1}{2}\left[\left(0 - \frac{-1}{6}\right) - (0 - 0)\right]$

$= \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$.

d) $X \sim Bin\left(30, \frac{1}{6}\right)$

$P(X \geq 2) = 1 - P(X < 2)$

$= 1 - [P(X = 0) + P(X = 1)]$

$= 1 - \left[30C_0 \left(\frac{5}{6}\right)^{30}\left(\frac{1}{6}\right)^0 + 30C_1 \left(\frac{5}{6}\right)^{29}\left(\frac{1}{6}\right)^1\right]$

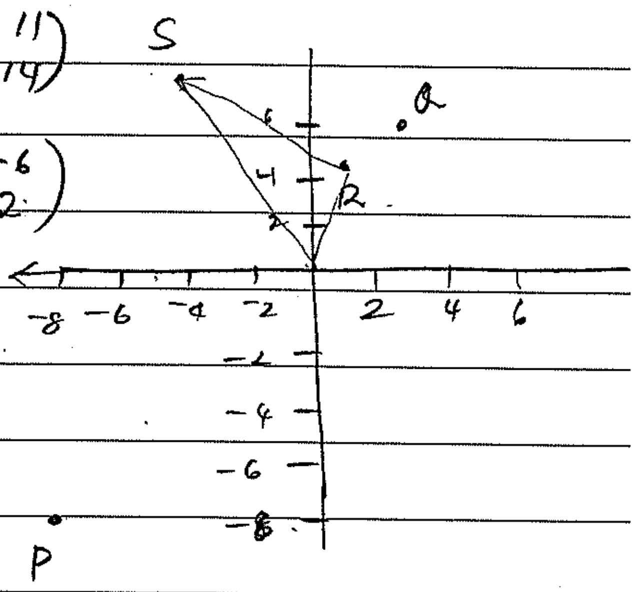
$= 0.97$

9) (i) $\overrightarrow{PQ} = \begin{pmatrix}8+3\\8+6\end{pmatrix} = \begin{pmatrix}11\\14\end{pmatrix}$

(ii) $\overrightarrow{RS} = \begin{pmatrix}-1-5\\-5+7\end{pmatrix} = \begin{pmatrix}-6\\2\end{pmatrix}$

(iii) $-\overrightarrow{PQ} - \overrightarrow{RS}$

$\begin{pmatrix}-11+6\\-14-2\end{pmatrix} = \begin{pmatrix}-5\\-16\end{pmatrix}$



Official Use only – Do NOT write anything, or make any marks below this line

Question 12

GLENWOOD

Name: ______________________

Teacher: ______________________

Question No.

☐ Year 11 ☐ Year 12

☐ Adv ☐ Ext 1 ☐ Ext 2

a)(i) $P(\text{atleast } 1) = P(X=1) + P(X=2)$

$= {}^2C_1(1-p)^1(p)^1 + {}^2C_2(1-p)^0(p)^2$

$= 2(1-p)(p) + p^2$

$= 2p - 2p^2 + p^2$

$= 2p - p^2$

(ii) $P(\text{atleast } 2) = P(X=2) + P(X=3)$

$= {}^3C_2(1-p)^1(p)^2 + {}^3C_3(1-p)^0(p)^3$

$= 3(1-p)p^2 + p^3$

$= 3p^2 - 3p^3 + p^3$

$= 3p^2 - 2p^3$

(iii) $2p - p^2 = 3(3p^2 - 2p^3)$

$2p - p^2 - 9p^2 + 6p^3 = 0$

$6p^3 - 10p^2 + 2p = 0$

$2p(3p^2 - 5p + 1) = 0$

$2p = 0$

$p = 0$

as $p > 0$

$\therefore p = \dfrac{5 \pm \sqrt{(-5)^2 - 4 \times 3 \times 1}}{2 \times 3}$

$= \dfrac{5 \pm \sqrt{13}}{6}$

$3p \quad -1$

$p \quad = 1$

-1

Since $0 < p < 1$ $\quad p = \dfrac{5 - \sqrt{13}}{6} \approx 0{\cdot}23$

b).

(i) ① $n=1$ is not proved.
② $n=k+1$ is simplified on both left & right sides simultaneously ③ $n=k+1$ used the assumption that $n=1$ is true.

(ii). prove the statement is true for $k=1$

LHS $2^0 = 1 - 1$

RHS $2^1 - 1 = 1$ LHS = RHS hence it's true for $k=1$.

Assume it's true for $n=k$

$2^0 + 2^1 + 2^2 + \ldots 2^{k-1} = 2^k - 1$

prove the statement is true for $n=k+1$

prove $2^0 + 2^1 + 2^2 + \ldots 2^k = 2^{k+1} - 1$

$\underbrace{2^0 + 2^1 + 2^2 + \ldots 2^{k-1}}_{S_k} + 2^k$

$2^k - 1 + 2^k$

$2 \times 2^k - 1$

$2^{k+1} - 1 = RHS$

Thus this is true for $n=k+1$ if it's true for $n=k$. By mathematical induction it's true for all integer values of k.

c) LHS $\operatorname{Cosec} x + \operatorname{Cot} x$

$= \frac{1+t^2}{2t} + \frac{1-t^2}{2t}$

$= \frac{1+t^2+1-t^2}{2t}$

$= \frac{2}{2t} = \frac{1}{t} = \frac{1}{\tan \theta/2} = \operatorname{Cot} \theta/2$

Official Use only – Do NOT write anything, or make any marks below this line.

$$\int_{\pi/3}^{\pi/2} (\operatorname{Cosec} x + \operatorname{Cot} x)\, dx$$

$$= \int_{\pi/3}^{\pi/2} \cot \frac{x}{2}$$

$$= \int_{\pi/3}^{\pi/2} \frac{\cos x/2}{\sin x/2} = 2\int_{\pi/3}^{\pi/2} \frac{\frac{1}{2} \times \cos x/2}{\sin x/2}$$

$$= 2\Big[\ln \sin x/2\Big]_{\pi/3}^{\pi/2}$$

$$= 2\Big[(\ln \sin \pi/4) - (\ln \sin \pi/6)\Big]$$

$$= 2\Big[\ln \frac{1}{\sqrt{2}} - \ln \frac{1}{2}\Big]$$

$$= 2\Big[\ln 2^{-\frac{1}{2}} - \ln 2^{-1}\Big]$$

$$= 2\Big[\frac{-1}{2} \ln 2 + \ln 2\Big]$$

$$= 2\Big[\frac{1}{2} \ln 2\Big]$$

$$= \ln 2.$$

Official Use only – Do NOT write anything, or make any marks below this line

GLENWOO

Name: Question 13 .

Teacher:

Question No.

☐ Year 11 ☐ Year 12

☐ Adv ☐ Ext 1 ☐ Ext 2

Question 13

a) (i) $X \sim Ber\left(\frac{18}{37}, 1\right)$ $\left(\frac{18}{37}, \frac{19}{37}\right)$

$\mu = \frac{18}{37}$

$\sigma^2 = \frac{18}{37} \times \frac{19}{37} = \frac{342}{1369}$

(ii) $Y \sim B\left(\frac{18}{37}, 5\right)$ $\left(5, \frac{18}{37}\right)$

$\mu = np = 5 \times \frac{18}{37} = \frac{90}{37}$

$\sigma^2 = npq = 5 \times \frac{18}{37} \times \frac{19}{37} = \frac{1710}{1369}$

(iii) $P(X=3) + P(X=4) + P(X=5)$

$= 5C_3\left(\frac{19}{37}\right)^2\left(\frac{18}{37}\right)^3 + 5C_4\left(\frac{19}{37}\right)^1\left(\frac{18}{37}\right)^4 + 5C_5\left(\frac{19}{37}\right)^0\left(\frac{18}{37}\right)^5$

$= 0.475$

b). (ii) $\text{Area} = 3\int_0^{2\pi} \sin x/_2 \, dx$

$= 3\left[\frac{-\cos x/_2}{1/2}\right]_0^{2\pi}$

$$= \left[-6\cos\left(\frac{x}{2}\right)\right]_0^{2\pi}$$

$$= (-6\cos\pi) - (-6\cos 0)$$

$$= 6 + 6$$

$$= 12\,u^2$$

$$V = \pi\int_0^{2\pi}\left(3\sin\frac{x}{2}\right)^2$$

$$= \frac{9\pi}{2} \times \int_0^{2\pi}(1 - \cos x)\,dx$$

$\cos 2x = 1 - 2\sin^2 x$

$2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$\sin^2 \frac{x}{2} = \frac{1}{2}(1 - \cos x)$

$$= \frac{9\pi}{2}\left[x - \sin x\right]_0^{2\pi}$$

$$= \frac{9\pi}{2}\left[(2\pi - \sin 2\pi) - (0 - \sin 0)\right]$$

$$= \frac{9\pi}{2}\left[(2\pi - 0) - (0)\right]$$

$$= \frac{9\pi}{2} \times 2\pi$$

$$= 9\pi^2\,u^3$$

c) Amount of liquid in tank $= 2500 + 20t - 15t$

$= 2500 + 5t$

Concentration of salt $= \dfrac{\text{Amount of salt}}{\text{Total amount of liquid}}$

$$= \frac{y}{2500 + 5t}$$

$C = g/L$

(ii) Rate of change of salt =
(Concentration of salt / litre) × Amount of liquid leaving

Use chain rule
$g/min = g/L \times L/m$

$= C \times -15 L/min = -15C$

(iii) $y' = -15C$

$\frac{dy}{dt} = -15 \times \frac{y}{2500+5t}$ ①

$\int \frac{dy}{-15y} = \frac{1}{5}\int \frac{5dt}{2500+5t}$

$-\frac{1}{15}\ln|y| = \frac{1}{5}\ln|2500+5t| + c$ ②

$\ln|y| = -\cancel{15}^{3} \times \frac{1}{\cancel{5}}\ln|2500+5t| + c$

$\ln|y| = \ln|2500+5t|^{-3} + c$

y & t are positive

$\ln y = \ln(2500+5t)^{-3} + c$

$e^{\ln y} = e^{\ln(2500+5t)^{-3} + c}$

$y = (2500+5t)^{-3} \times e^{c}$

$y = A(2500+5t)^{-3}$

At $t=0$, $y = 25$ ③

$25 = A(2500+5\times 0)^{-3}$

Official Use only – Do NOT write anything, or make any marks below this line

$A = 25 \times 2500^3$

$y = \dfrac{25 \times 2500^3}{(2500+5t)^3}$

(iv) Amount of water in tank $= 2500 + 20t - 15t$

$= 2500 + 5t$.

Overflow when

$5000 = 2500 + 5t$

$2500 = 5t$

$t = \dfrac{2500}{5} = 500$

Amount of salt at $t = 500$.

$y = \dfrac{25 \times 2500^3}{2500 + 5 \times 500}$

$= 3.125\text{kg}$.

Question 14

Name: ____________

Teacher: ____________

Question No.

☐ Year 11 ☐ Year 12

☐ Adv ☐ Ext 1 ☐ Ext 2

a) $y' = \dfrac{6}{5x^2+4x-1}$

(i) $\dfrac{5}{5x-1} - \dfrac{1}{x+1} = \dfrac{5(x+1) - 1(5x-1)}{(5x-1)(x+1)}$

$= \dfrac{5x + 5 - 5x + 1}{(5x-1)(x+1)}$

$= \dfrac{6}{(5x-1)(x+1)} = \dfrac{6}{5x^2+4x-1}$

(ii) $\dfrac{dy}{dx} = \dfrac{6}{5x^2+4x-1}$

$\int \dfrac{dy}{dx} = \int \dfrac{5}{5x-1} - \int \dfrac{1}{x-1}$

$y = \ln(5x-1) - \ln((x-1)) + C$

$y = \ln \dfrac{5x-1}{x-1} + C$

when $x = \frac{1}{2}$, $y = 3$

$$3 = \ln\frac{\left(\frac{5}{2}-1\right)}{\left(\frac{1}{2}+1\right)} + c$$

$$3 = \ln\left(\frac{\frac{3}{2}}{\frac{3}{2}}\right) + c$$

$$3 = \ln 1 + c$$

$$3 = 0 + c$$

$$y = \ln\left(\frac{5x-1}{x+1}\right) + 3$$

b) $s = 110t\cos\theta\,\underset{\sim}{i} + (110t\sin\theta - 4.9t^2)\,\underset{\sim}{j}$

Max horizontal range when $y = 0$

$$110t\sin\theta - 4.9t^2 = 0$$

$$t(110\sin\theta - 4.9t) = 0$$

$t = 0$ | $110\sin\theta - 4.9t = 0$

$$4.9t = 110\sin\theta$$

$$t = \frac{110\sin\theta}{4.9}$$

when $t = \frac{110\sin\theta}{4.9}$

$$x = 110 \times \frac{110\sin\theta}{4.9} \times \cos\theta$$

$$x = \frac{12100\sin\theta\cos\theta}{4.9}$$

$$x = \frac{6050 \times 2\sin\theta\cos\theta}{4.9} = \frac{6050 \times \sin 2\theta}{4.9}\,\frac{\times 2}{\times 2} = \frac{12100\sin 2}{9.8}$$

(ii) $400 < Range < 450$

$400 < \frac{12100 \sin 2\theta}{9.8} < 450$

$3920 < 12100 \sin 2\theta < 4410$

$\frac{196}{605} < \sin 2\theta < \frac{441}{12100}$

$18^\circ 54' < 2\theta < 21^\circ 22'$

$9^\circ 27' < \theta < 10^\circ 41'$

(iii) At $t = 3.4$, $y = 8$

$y = 110 t \sin\theta - 4.9 t^2$

$8 = 110(3.4)\sin\theta - 4.9(3.4)^2$

$8 = 374 \sin\theta - 56.644$

$\sin\theta = \frac{8 + 56.644}{374}$

$\theta = 9^\circ 57'$

θ is between $9^\circ 27'$ and $10^\circ 41'$

∴ the ball would have made it to green.

c) ~~If~~ $y = \cos^{-1}(\sin x)$

$$\frac{dy}{dx} = \frac{-\cos x}{\sqrt{1-\sin^2 x}}$$

$$= \frac{-\cos x}{\sqrt{\cos^2 x}}$$

$$= \frac{-\cos x}{\pm\cos x} = \pm 1$$

(ii) Since the gradient is a contant which is not zero the function has no stationay points.

(iii) Domain $(-\infty, \infty)$

$-1 \leq \sin x \leq 1$

when $\sin x = -1$

$\cos^{-1}(-1) = \pi$

when $\sin x = +1$

$\cos^{-1}(1) = 0$

∴ Range $[0, \pi]$

$\cos^{-1}(\sin 0)$

$\cos^{-1}(\sin \pi/2)$

$\cos^{-1}(\sin \pi)$

$\cos^{-1}(\sin 3\pi/2)$

$\cos^{-1}(\sin (2\pi))$